Name: _____ Date: _____

A2CC Reciprocal Trig Functions

We have now seen three primary trigonometric functions, the sine, cosine, and tangent functions. Each of these can be defined in terms of either **ratios of the sides of a right** triangle or **the unit circle**. For each of these functions, though, there exists what is known as a **reciprocal function**. Their definitions are shown below.

**THE OTHER FOUR TRIGONOMETRIC FUNCTIONS**

**1. SECANT:** $\sec(\theta)=\dfrac{1}{\cos(\theta)}$ 2. **COSECANT:** $\csc(\theta)=\dfrac{1}{\sin(\theta)}$

3. **COTANGENT:** $\cot(\theta)=\dfrac{1}{\tan(\theta)}$ or equivalently $\cot(\theta)=\dfrac{\cos(\theta)}{\sin(\theta)}$

***Exercise* #1:** Considering your work with sine and cosine, evaluate each of the following. Express your answers in exact and simplest form.

(a) $\sec\left(60^\circ\right)$ (b) $\cot\left(150^\circ\right)$ (c) $\csc\left(\dfrac{3\pi}{4}\right)$

***Exercise* #2:** Which of the following is closest to the value of $\sec\left(52^\circ\right)$?

(1) 0.62 (3) 0.36

(2) 1.62 (4) 2.48

Because each of these reciprocal trigonometric functions has a variable denominator, there will be angles at which these denominators are zero and hence the function is undefined.

***Exercise* #3:** Which of the following values of *x* is *not* in the domain of $y=\csc(x)$?

(1) $x=180^\circ$ (3) $x=90^\circ$

(2) $x=60^\circ$ (4) $x=135^\circ$

Because each of these functions is dependent on sine and/or cosine, it is possible to determine the **sign** (positive or negative nature) of each based on the quadrant of the input angle.

***Exercise* #4:** Determine the sign of each of the following trigonometric functions in the quadrant specified.

(a) $\cot(\beta)$ for $\beta$ in quad. II (b) $\sec(\beta)$ for $\beta$ in quad. IV (c) $\csc(\beta)$ for $\beta$ in quad. III

***Exercise* #5:** If $\cot(\theta) < 0$ and $\sec(\theta) > 0$ then $\theta$ could be which of the following angles?

(1) $\theta = 48^\circ$ (3) $\theta = 122^\circ$

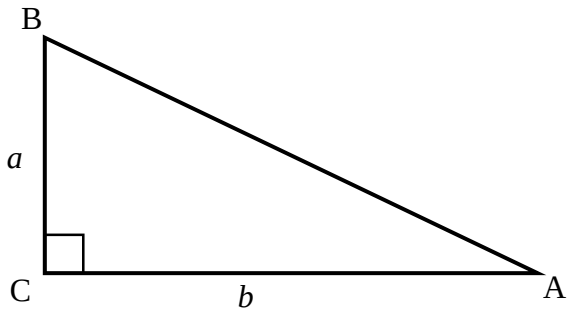
(2) $\theta = 310^\circ$ (4) $\theta = 225^\circ$

---

We should also be able to produce all of the trigonometric ratios (all SIX of them) if we are given a right triangle.

***Exercise* #6:** A right triangle is shown below with sides of length *a* and *b*.

(a) Find the length of the hypotenuse in terms of *a* and *b*. Label on the diagram.




(b) State the value of each of the following trigonometric ratios in terms of the constants *a* and *b*.

$\sin A =$ $\csc A =$

$\cos A =$ $\sec A =$

$\tan A =$ $\cot A =$

***Exercise* #7:** If $\alpha$ is an angle whose terminal ray lies in the fourth quadrant and $\cos\alpha = \frac{1}{3}$, then determine the exact value of $\csc\alpha$. Show how you arrived at your answer.

Name: _____ Date: _____

A2CC Reciprocal Trig Functions

## HOMEWORK

### FLUENCY

1. Determine the value of each of the following in exact and simplest form (leave no complex fractions).

   (a) $\csc\left(30^\circ\right)$ (b) $\cot\left(90^\circ\right)$ (c) $\sec\left(180^\circ\right)$

   (d) $\cot\left(\frac{\pi}{3}\right)$ (e) $\csc\left(\frac{3\pi}{2}\right)$ (f) $\sec\left(\frac{5\pi}{4}\right)$

2. Use your calculator to determine the value of each of the following to the nearest *hundredth*.

   (a) $\cot\left(115^\circ\right)$ (b) $\sec\left(312^\circ\right)$ (c) $\csc\left(245^\circ\right)$

3. In simplest radical form, $\sec\left(135^\circ\right)$ is equal to

   (1) $-\frac{\sqrt{2}}{3}$ (3) $-\frac{\sqrt{2}}{2}$

   (2) $-\sqrt{2}$ (4) $-\frac{\sqrt{3}}{2}$ _____

4. Which of the following is nearest to the value of $\cot\left(220^\circ\right)$?

   (1) 1.19 (3) –2.74

   (2) 3.17 (4) –0.85 _____

5. For which of the following values of $\alpha$ is $\cot(\alpha)$ undefined?

(1) $60^\circ$ (3) $180^\circ$

(2) $90^\circ$ (4) $135^\circ$

6. For which angle, $\beta$, below will $\sec(\beta)$ not exist?

(1) $30^\circ$ (3) $180^\circ$

(2) $45^\circ$ (4) $90^\circ$

7. Determine whether each function in the tables below is positive, $(+)$, or negative, $(-)$, for angles whose terminal rays lie in the respective quadrants. Use the table in part (a) to help create the table in (b).

(a)

| | I | II | III | IV |
|---|---|---|---|---|
| $\cos(\theta)$ | | | | |
| $\sin(\theta)$ | | | | |

(b)

| | I | II | III | IV |
|---|---|---|---|---|
| $\tan(\theta)$ | | | | |
| $\cot(\theta)$ | | | | |
| $\sec(\theta)$ | | | | |
| $\csc(\theta)$ | | | | |

8. For the angle $\beta$ it is known that $\csc(\beta) > 0$ and $\sec(\beta) < 0$. When drawn in standard position, the terminal ray of $\beta$ lies in quadrant

(1) I (3) III

(2) II (4) IV

9. The angle $\theta$ when drawn in standard position has its terminal ray in the second quadrant. If it is known that $\sin\theta = \frac{5}{13}$ then determine the values of all of the remaining trigonometric functions.

(a) $\cos\theta$ (b) $\tan\theta$ (c) $\sec\theta$

(d) $\csc\theta$ (e) $\cot\theta$